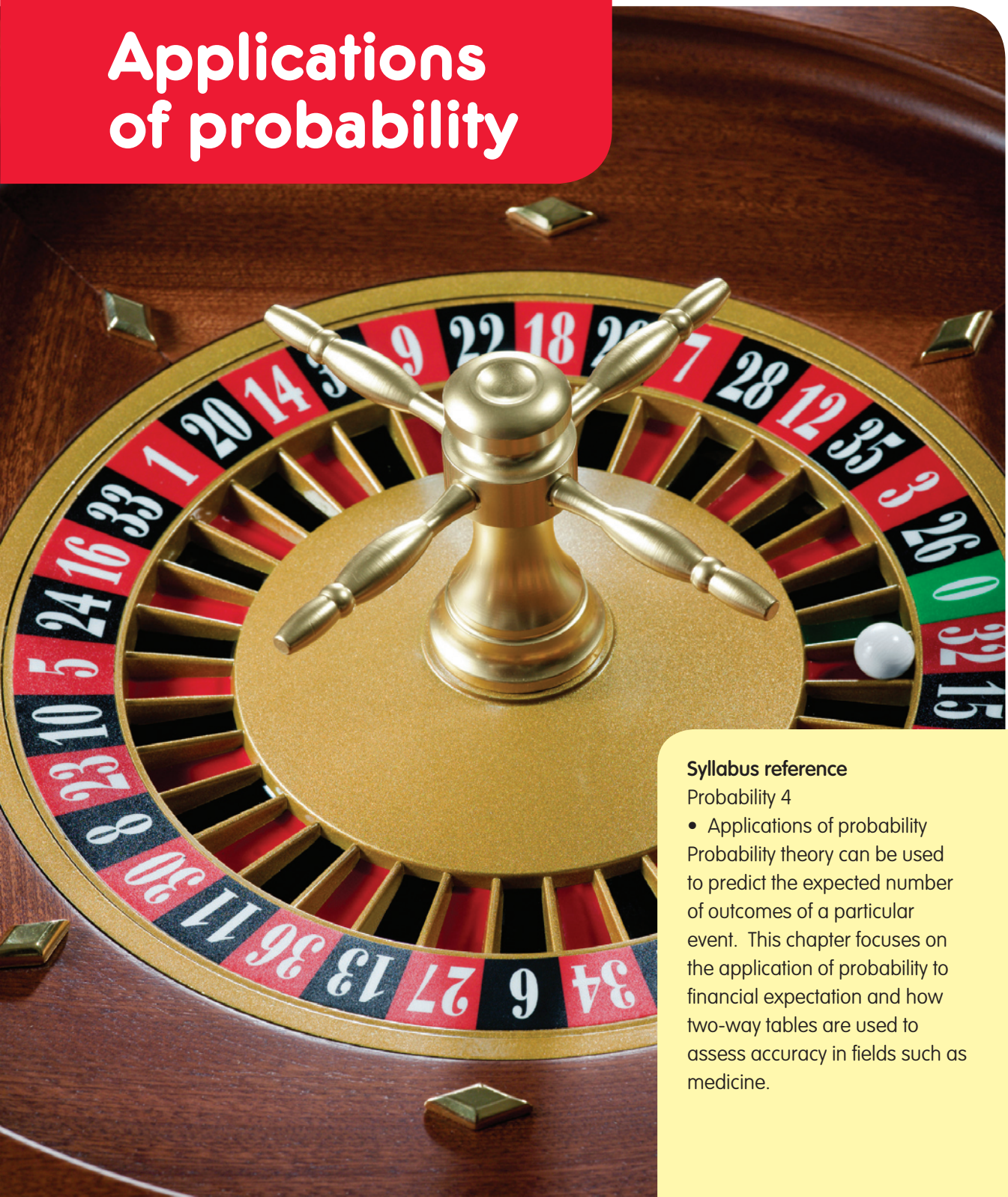
7

Applications of probability



Syllabus reference

Probability 4

- Applications of probability

Probability theory can be used to predict the expected number of outcomes of a particular event. This chapter focuses on the application of probability to financial expectation and how two-way tables are used to assess accuracy in fields such as medicine.

ARE YOU READY?

Try the questions below. If you have difficulty with any of them, extra help can be obtained by completing the matching SkillSHEET. Either click on the SkillSHEET icon next to the question on the *Maths Quest HSC Course* eBookPLUS or ask your teacher for a copy.

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Single event probability

Single event probability

1 Calculate the probability of each of the following.
- **a** Rolling a die and getting a number greater than 2.
- **b** Winning a raffle after purchasing 10 tickets and knowing there are 500 tickets in the draw.
- **c** Selecting an even number from the numbers 1 to 99 inclusive.

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Tree diagrams

Tree diagrams

2 Two of the digits 3, 5, 6 and 7 are used to form a two-digit number such that no digit can be repeated. Draw a tree diagram to list all possible two-digits numbers that can be formed.

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Probability trees

Probability trees

3 In any given hour of television there are 12 minutes of advertisements. If Tony turns the television on at two randomly selected times between 7.00 pm and 8.00 pm.
- **a** use a probability tree to show all possible outcomes
- **b** calculate the probability that on both occasions Tony turns on the television during an advertisement.

7A Expected outcomes

Suppose that we toss a coin 100 times. How many times would you expect the coin to land Heads? As each outcome is equally likely, we would expect there to be 50 Heads and 50 Tails. How can this be shown to be true?

The number of times that we expect a certain outcome to occur is found by multiplying the probability of each outcome by the number of trials. In the above case, the probability of the coin landing heads is $\frac{1}{2}$, and this is multiplied by the number of trials (100). The result is an expectation of 50 Heads in 100 tosses of the coin.

The **expected outcome** is the number of times that we expect a particular outcome to occur in a certain number of trials.

WORKED EXAMPLE 1

A die is rolled 120 times. How many 6s would you expect to occur in 120 rolls of the die?

THINK	WRITE
1 Calculate the probability of rolling a 6.	$P(\text{six}) = \frac{1}{6}$
2 Multiply the probability of a 6 by the number of trials.	Expected number of 6s $= \frac{1}{6} \times 120$ $= 20$

If the expected number of 6s is 20 in 120 rolls of a die, this does not mean that this is what will occur. It may be that on one occasion we may get 25 sixes in 120 rolls, another occasion we may get only 10 sixes. However, we expect that if we repeat the experiment often enough, we would get an average of 20 sixes in 120 rolls.

Rolling a die

1. Each person is to take a die and roll it 120 times and record the number of 6s rolled.
2. What is the most number of 6s rolled by anyone in 120 rolls of the die?
3. What is the least number of 6s rolled by anyone in 120 rolls of the die?
4. What is the average number of 6s rolled by the class in 120 rolls of the die? How does this compare with the expected outcome of 20?

The expected outcome does not need to be a whole number. In many cases this will not be so. Consider the example below.

WORKED EXAMPLE 2

Roger draws a card from a standard deck, notes the suit and replaces the card in the deck. If Roger repeats this process 50 times, how many spades can Roger expect to have drawn?

THINK	WRITE
1 Calculate the probability of drawing a spade.	$P(\text{spade}) = \frac{1}{4}$
2 Calculate the expected number of spades by multiplying the probability by the number of trials.	Expected number of spades $= \frac{1}{4} \times 50$ $= 12.5$

Obviously, after drawing 50 cards, Roger could not have drawn 12.5 spades. The number of spades drawn must of course be a whole number. However, if this experiment were repeated a number of times, we would expect to have drawn an *average* of 12.5 spades in every 50 cards.

The expected outcome method can be applied to any probability experiment. This includes multistage events in which it may be necessary to draw a tree diagram or probability tree to calculate the probability of a particular outcome.

WORKED EXAMPLE 3

A psychologist is conducting a study on the upbringing of boys. For the study, the psychologist selects 100 couples with exactly three children. How many of these couples would the psychologist expect to have three boys?

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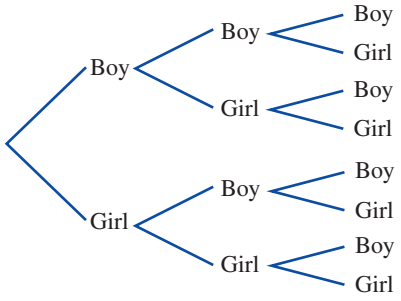
Worked example 3

THINK

1. Draw a tree diagram showing the sample space for three children.

WRITE



2. Calculate the probability of three boys.

$P(\text{three boys}) = \frac{1}{8}$

3. Calculate the expected number by multiplying the probability of three boys by the number of couples in the study.

Expected number of couples with three boys
$= \frac{1}{8} \times 100$
$= 12.5$

REMEMBER

1. The number of times an event can be expected to occur in a number of trials is calculated by multiplying the probability of that event by the number of trials.
2. The number of times we expect an event to occur does not mean the event will occur that number of times. Rather, this is the average number of times we would expect this event to occur.

EXERCISE
7A Expected outcomes

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Single event probability

1 WE1 Calculate the number of times that a coin can be expected to land Tails in 40 tosses.

2 A die is rolled 300 times. Calculate the expected number of 6s to be rolled.

3 A card is drawn from a standard deck, its suit is noted and the card is replaced in the deck. Calculate the expected number of hearts in 100 selections.

4 A barrel contains five red marbles, four blue marbles and a green marble. A marble is drawn from the barrel. Its colour is noted, and it is then replaced in the barrel. In 70 selections from the barrel, how many times would we expect to select:

a a red marble? **b** a blue marble? **c** a green marble?

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Tree diagrams

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Probability trees

5 WE2 Lorna spends a night at the greyhounds. There are 10 races, and in each race there are eight greyhounds. Lorna bets on number 5 in every race. Calculate the number of winning greyhounds that Lorna can expect to back.

6 A card is drawn from a standard deck; the card is then noted and replaced in the deck. This is repeated 100 times. Calculate the number of times (where necessary, correct to 2 decimal places) that we could expect to select:
- **a** a club
- **b** a red card
- **c** an ace
- **d** a court card (ace, king, queen or jack)
- **e** the king of diamonds.

7 Kevin buys a ticket in a meat raffle every week. There are 100 tickets and four prizes.
- **a** Calculate the probability of Kevin winning a prize in the raffle.
- **b** How many prizes can Kevin expect to win in one year?

8 Janice buys a ticket in every lottery. In each lottery there are 180 000 tickets, a first prize and 3384 cash prizes. One lottery is drawn every weekday for 52 weeks a year. Calculate the number of times in 10 years that Janice can expect to win:
- **a** first prize (as a decimal, correct to 3 significant figures)
- **b** a cash prize (as a decimal, correct to 3 significant figures).

9 MC A meeting is attended by 350 men and 150 women. At the meeting 100 people will be chosen to make a speech. What is the expected number of women to make speeches?

A 15 **B** 30 **C** 50 **D** 150

10 MC A tennis club runs a raffle each week with 100 tickets. Fumiko buys one ticket each week. The expected number of raffle wins over a period of 50 weeks is:

A 0.01 **B** 0.5 **C** 1 **D** 20

11 WE3 Four coins are tossed simultaneously in the air. If this were repeated 80 times, on how many occasions would you expect the coins to land with four Heads?

12 The digits 2, 5, 6, 7 and 9 are written on cards and placed face down. Three are then chosen and arranged to form a three-digit number. If this is repeated 150 times, what is the expected number of:
- **a** odd numbers?
- **b** numbers greater than 600?
- **c** multiples of five?

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Die rolling

13 Two dice are rolled 100 times. Copy and complete the table below to calculate the expected number of occurrences of each total in 100 rolls of the dice. Give each answer correct to 1 decimal place.

Outcome	2	3	4	5	6	7	8	9	10	11	12
Probability											
Expected no.											

14 A barrel contains 15 blue marbles and 5 red marbles. Two marbles are selected from the barrel, the first not being replaced in the barrel before the second is chosen. This experiment is repeated 100 times. On how many occasions (correct to 2 decimal places) would you expect the two marbles chosen to be:
- **a** both blue?
- **b** both the same colour?
- **c** different colours?
- **d** selected with at least one being blue?

Further development

15 A die is biased as shown in the table below. What average outcome would you expect for a large number of rolls of this die?

Number	Probability
1	0.2
2	0.2
3	0.2
4	0.2
5	0.1
6	0.1

16 Anthony is a basketball player and has a probability of 0.7 of shooting a basket from the free-throw line. During a match Anthony goes to the free-throw line 10 times. On each occasion he receives two free throws.

a What is the expected number of successful free throws that Anthony will make?

b What is the expected number of times that Anthony will be successful with both free throws?

17 Eldrick is a golfer and is playing the par three 17th hole. Eldrick has a 80% chance of hitting his first shot onto the green and a 45% of sinking a putt from any point on the green.

a Find the probability that Eldrick is able to get the ball in the hole in two shots. (This score of one under par is called a 'birdie'.)

b If Eldrick plays the 17th hole four times during a tournament, what is the expected number of birdies?

18 Every Friday night a local club runs a meat raffle. In the raffle there are 1000 tickets, and Rhonda buys 5 tickets. There are 70 prizes in the raffle. Find the number of prizes that Rhonda can expect to win:

a in one night

b over a year if she buys the same number of tickets each week.

19 A criminal gang has the technology to copy peoples ATM cards. They do not have the technology to identify people's PINs. If they try to use the card they can have three attempts at the PIN before the bank will deactivate the card.

a What is the probability that the gang is able to guess the 4-digit PIN?

b How many cards does the gang need to copy before they can expect to be able to guess one PIN?

20 A telephone insurance salesperson has a 0.16 probability of being able to sell an insurance policy. How many telephone calls does the salesperson need to make so they can expect to sell 60 insurance policies?

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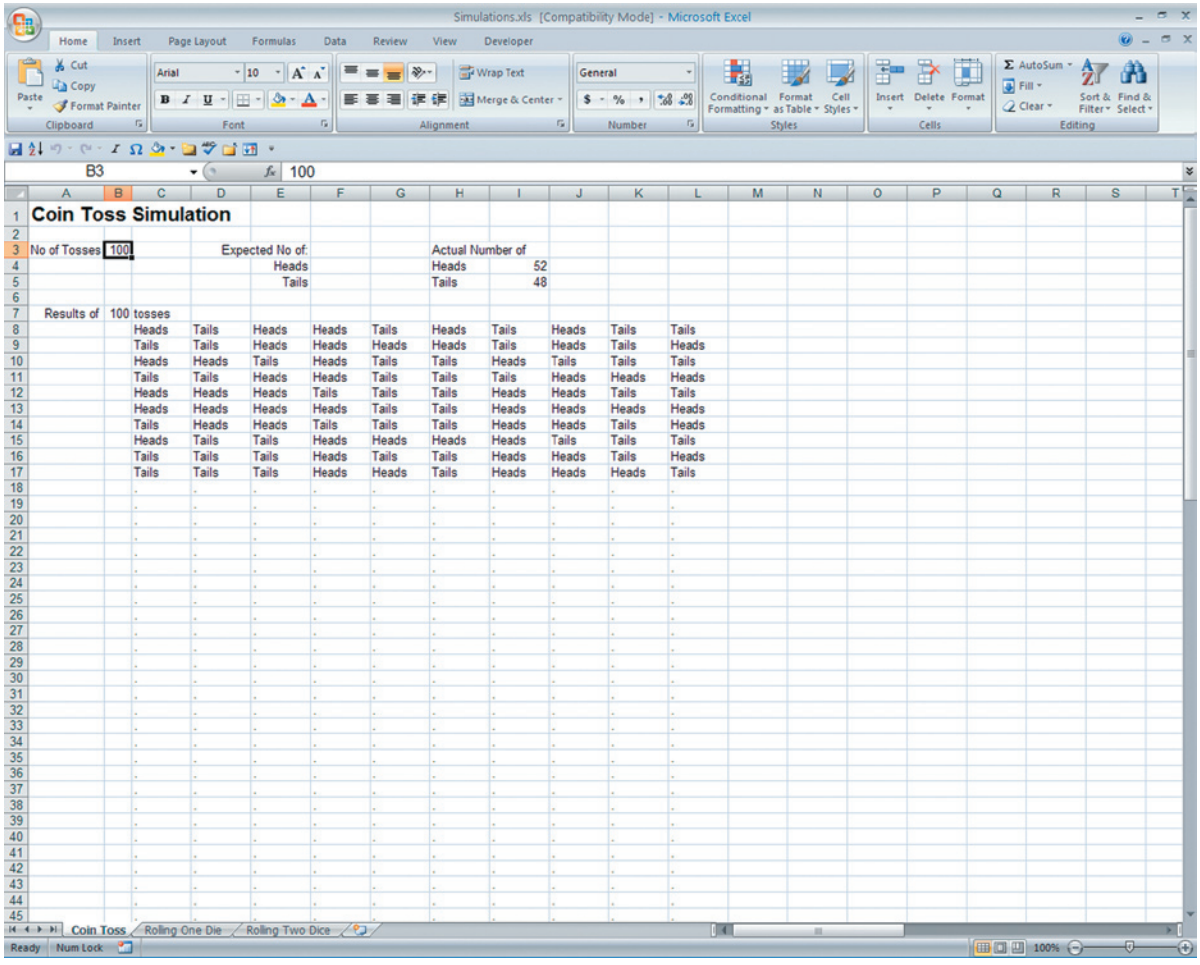
Simulations

Computer Application 1: Simulations

A **simulation** is where a computer gives results to an experiment that are similar to those that would occur if the experiment were actually performed. For example, if a coin is tossed 100 times, a computer can randomly choose Heads or Tails in a fraction of a second. In each case, the probability of each outcome is $\frac{1}{2}$ and we are saved the process of actually tossing the coin.

1. Access the spreadsheet *Simulations* from your *Maths Quest General Mathematics HSC Course* eBookPLUS.



2. The first worksheet has a coin toss simulation. In cell **B3** enter the number of times you wish to toss the coin, in cell **F4** enter the expected number of heads and in cell **F5** enter the expected number of tails.
3. How do the simulation results compare with the expected outcome? Complete 10 simulations and average the results. Is this answer closer to the expected number of outcomes that you have calculated?
4. Repeat this process for each one of the other simulations on rolling a die and rolling two dice.

7B Financial expectation

We can use expected outcomes to make an assessment of financial situations where probability is concerned. In particular, this applies to many forms of gambling. The average financial outcome from such a situation is called the **financial expectation**.

Consider a simple game where two people are betting \$1 on the toss of a coin. The probability of winning the toss is $\frac{1}{2}$ and this will give a financial return of \$1, while the probability of losing the game is $\frac{1}{2}$ and this will lead to a financial loss of \$1. We need to consider a financial loss as being negative.

To calculate the financial expectation, we multiply each financial outcome by the probability of that outcome and then add the results together. In the above example:

$$\text{Financial expectation} = \frac{1}{2} \times \$1 + \frac{1}{2} \times -\$1$$
$$= \$0$$

This financial expectation tells us that we can expect to neither gain nor lose money in this game over a long period of time. This does not mean that this will be the outcome, but it is the average expected outcome.

WORKED EXAMPLE 4

A game is played where a die is rolled. If a six is rolled, the player wins $6; if a five is rolled, the player wins $3; and if any other number is rolled, the player loses $3. What is the financial expectation from this game?

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Worked example 4

THINK

Financial expectation is calculated by multiplying the financial result of each outcome by the probability of each outcome and adding the results together.

WRITE

$$\text{Financial expectation} = \frac{1}{6} \times -\$3 + \frac{1}{6} \times -\$3 + \frac{1}{6} \times -\$3 + \frac{1}{6} \times -\$3 + \frac{1}{6} \times \$3 + \frac{1}{6} \times \$6$$
$$= -\$0.50$$

In worked example 4, the financial expectation is negative. This means that over an extended period of time we can expect to lose 50c per game.

This type of calculation can be applied to other financial situations such as the share market.

WORKED EXAMPLE 5

Over the past 10 years the price of a particular share has risen by $2 on five occasions, by $1 on two occasions and has fallen by $3 on three occasions. What is the financial expectation for this share price in the next year?

THINK

1. Calculate the (experimental) probability of each outcome.
2. Calculate the financial expectation using the experimental probabilities.

WRITE

$\text{P(\$2 profit)} = \frac{5}{10}$, $\text{P(\$1 profit)} = \frac{2}{10}$,
$\text{P(\$3 loss)} = \frac{3}{10}$

$$\text{Financial expectation} = \frac{5}{10} \times \$2 + \frac{2}{10} \times \$1 + \frac{3}{10} \times -\$3$$
$$= \$0.30$$

In this example, where the financial expectation is positive, we can expect to make a profit. Again this does not mean we will make a profit but the average share price fluctuation is a gain of 30c.

REMEMBER

1. Financial expectation is the average return in a financial situation.
2. The financial expectation is calculated by multiplying each possible financial outcome by the probability of that financial outcome and adding the results together.
3. A financial loss is indicated by a negative financial outcome while a financial gain is a positive financial outcome.

EXERCISE

7B Financial expectation

1 WE4 A game is played where a die is rolled. If a 1 or a 6 is rolled, the player wins $2; if any other number is rolled, the player loses $1. What is the financial expectation from this game?

2 There are five cards labelled 1, 2, 3, 4 and 5. A card is selected. If it is even, you win $5, and if it is odd, you lose $4. Calculate the financial expectation.

3 Soon-Jung plays a game in which two coins are tossed. If he throws two Heads, he wins $5; if he throws two Tails, he loses $3. For one Head and one Tail, he loses $2. Calculate the financial expectation from this game.

4 In a card game, the player selects a card from a standard deck. The player then wins $5 for an ace and $2 for a king, queen or jack. If any other card is selected, $1 is lost. Calculate the financial expectation from this game.

5 A raffle has 1000 tickets that sell for $1 each. There is a first prize of $400, a second prize of $200 and a third prize of $100. Calculate the financial expectation from the purchase of one ticket in the raffle.

6 WE5 Over the past 20 years shares in the company FIA have increased by $5 on eight occasions, increased by $2 on six occasions and fallen by $3 on six occasions. Calculate the financial expectation for a person who buys FIA shares for the coming year.

7 Look at the roulette wheel on the right.

a How many slots are on the wheel?

b How many of these slots are:

i black?

ii red?

iii green?

c Francis bets $10 on black. If a black number is spun, he wins $10; otherwise, he loses $10. Calculate Francis's financial expectation.

8 MC A game is played where a die is rolled.
The cost of the game is $1. The players are returned their $1 plus an extra $5 if they can roll a 6.
The financial expectation from this game is:

A 0 **B** 0.17 **C** –0.17 **D** –1

9 MC Which of the following games has the greatest financial expectation?

A A coin is tossed. Players win $1 if they toss a Head; otherwise, $1 is lost.

B Two coins are tossed. Players win $2 if they toss two Heads; otherwise, $1 is lost.

C A die is rolled. The player wins $6 for rolling a 6; otherwise, $1 is lost.

D Two dice are rolled. The player wins $6 for rolling a total of six; otherwise, $1 is lost.

10 In a dice game, two dice are rolled.

- The player wins $1 for rolling a total of 7 or 11.
- The player loses $1 for rolling a total of 2, 3 or 12.
- If any other total is rolled, the dice are rolled again.

What is the financial expectation from this game?

11 In the Jackpot lottery there are 180 000 tickets sold at $2 each. The prizes are shown below.

1st prize $100 000	2nd prize $10 000	3rd prize $5000
2 prizes of $1000	2 prizes of $500	5 prizes of $200
12 prizes of $100	60 prizes of $50	600 prizes of $20
2700 prizes of $10		

Calculate the financial expectation from purchasing a $2 lottery ticket.

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Further development

12 **a** A game is played where the financial expectation is 0.2. Explain what this means.

b A game is played where the financial expectation is –0.2. Explain what this means.

13 Over the past 10 years the share price in a company has risen by $5 in three of the years and has fallen by $1.50 in the other seven years. Based upon these results, if I purchase shares in this company, what would be my financial expectation for the year ahead?

14 The following bet is suggested to you:
Roll the die (6-sided) and if it:

a shows an even number, you get $10
b shows a 5, you pay $30
c shows a 3, you pay $15
d shows a 1, nothing happens.

What is the expected loss or gain for this bet?

15 If you are given one of 250 tickets in a raffle that has a prize of $400, what is the value of the ticket?

16 In the game of Keno, 20 numbers are drawn out of 80. The player can choose to match between 1 and 10 numbers.
Note: The function ${}^{n}C_{r}$ is a standard mathematical function. It can be found in the probability menu for scientific, graphics or CAS calculators. It is useful when evaluating large numbers. ${}^{n}C_{r}$ represents the number of unordered selections of r objects from n available objects.

a The probability of winning the two-number game is $\frac{{}^{20}C_2}{{}^{80}C_2} \approx 0.06$. The prize for the two-number game is $12.

i What is the expected return for this game?
ii Given that the game costs $1 to play, what is the financial expectation for the player.

b Copy and complete the table below.

Numbers selected	Prize	Probability of winning	Financial expectation
1	$3	0.25	–0.25
2	$12	0.06	
3	$43		
4	$112		
5	$610		
6	$1500		

c Based on the above table which game between 1 and 6 numbers is best value for the player.

17 In Keno the 7-, 8-, 9- and 10-number games do not have a fixed prize; rather, the prize jackpots until won.

a The probability of winning the 7-number game is $\frac{{}^{20}C_7}{{}^{80}C_7} \approx 2.44 \times 10^{-5}$. Solve the equation $2.44 \times 10^{-5} - P = 1$ to find the value of the minimum prize needed so that the player has a positive financial expectation.

b Copy and complete the table below.

Numbers selected	Probability of winning	Minimum prize for a positive financial expectation
7	2.44×10^{-5}	
8		
9		
10		

7C Two-way tables

A **two-way table** is a two-dimensional grid that shows the outcome of an experiment in terms of two variables. A two-way table is used to display information and allows for predictions to be made based on this information.

Consider an example where 400 newborn babies are tested for a genetic condition. The two-way table below displays these results.

	Test results		Total
	Accurate	Not accurate	
With condition	85	9	94
Without condition	304	2	306
Total	389	11	

The information that is given to us by this two-way table is that:

- 94 babies have the condition of which 85 were diagnosed and 9 were not
- 306 babies did not have the condition of which 304 were shown not to have the condition by the test and 2 who were told they had the condition but they did not (these are known as false positives).

From such a two-way table we can tell the total number of babies with and without the condition and the total number of correct and incorrect diagnoses made.

WORKED EXAMPLE 6

A new test was designed to assess the reading ability of students entering high school. The results were used to determine if the students' reading level was adequate to cope with high school. The students' results were then checked against existing records.

- **150 adequate readers sat for the test and 147 of them passed.**
- **50 inadequate readers sat for the test and 9 of them passed.**

Present this information in a two-way table.

THINK

Draw up the table showing the number of students whose reading was adequate and the number of students for whom the results of the new test were confirmed.

WRITE

	Test results		Total
	Passed	Did not pass	
Adequate readers	147	3	150
Inadequate readers	9	41	50
Total	156	44	

When information is presented in a two-way table, conclusions can be made about the accuracy of such a test and calculations can be made about the probability that such a test is accurate.

WORKED EXAMPLE 7

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Worked example 7

A batch of sniffer dogs is trained by customs to smell drugs in suitcases. Before they are used at airports they must pass a test. The results of that test are shown in the two-way table below.

	Test results		
	Detected	Not detected	Total
No. of bags with drugs	24	1	25
No. of bags without drugs	11	164	175
Total	35	165	

a How many bags did the sniffer dogs examine?
b In how many bags did the dogs detect drugs?
c In what percentage of bags without drugs did the dogs incorrectly detect drugs?
d Based on the above results, what is the probability that the dogs will not detect a bag carrying drugs?

THINK

a Add both total columns; they should give the same result.

b The total of the detected column.

c There were 175 bags without drugs but dogs incorrectly detected them in 11 bags. Write this as a percentage.

d Use the probability formula. Of 25 bags with drugs, 1 went undetected.

WRITE

a 200 bags were examined.

b The dogs detected drugs in 35 bags.

c Percentage incorrectly detected

$= \frac{11}{175} \times 100\%$

$= 6.3\%$

d P(bag going undetected) $= \frac{1}{25}$

As a result of studying a two-way table, we should also be able to make judgements about the information given in the tables. In the above worked example only one bag out of 25 with drugs went undetected. Although the dogs incorrectly detected drugs in 11 bags that did not have them, they still have an overall accuracy of 94%.

Many two-way tables will require you to make your own value judgements about the conclusions established by the test. For example, the 94% overall accuracy recorded above may be considered very acceptable.

REMEMBER

1. A two-way table is used to display test results and examine the accuracy of these results.
2. The table displays horizontally the numbers with and without a certain condition, and vertically displays information about accuracy.
3. The table can be used to make calculations about the accuracy of the test and about the probability of those test results being accurate in an individual case.

EXERCISE

7C Two-way tables

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Two-way frequency tables

1 WE6 A test is developed to test for the flu virus. To test the accuracy, 500 people are tested.
- 100 people who are known to have the flu are tested and the test returns 98 positive results.
- 400 people who are known not to be infected with the virus are tested with 12 false positives being returned.

Display this information in the two-way table below.

	Test results		Total
	Accurate	Not accurate	
With virus			
Without virus			
Total			

2 One thousand people take a lie detector test. Of 800 people known to be telling the truth, the lie detector indicates that 23 are lying. Of 200 people known to be lying, the lie detector indicates that 156 are lying. Present this information in a two-way table.

3 WE7 The two-way table shown below displays the information gained from a medical test screening for a virus. A positive test indicates that the patient has the virus.

	Test results		Total
	Accurate	Not accurate	
With virus	45	3	48
Without virus	922	30	952
Total	967	33	

a How many patients were screened for the virus?
b How many positive tests were recorded? (that is, in how many tests was the virus detected?)
c What percentage of test results were accurate?
d Based on the medical results, if a positive test is recorded, what is the probability that you actually have the virus?

4 The two-way table below indicates the results of a radar surveillance system. If the system detects an intruder, an alarm is activated.

	Test results		Total
	Alarm activated	Not activated	
Intruders	40	8	48
No intruders	4	148	152
Total	44	156	

a Over how many nights was the system tested?
b On how many occasions was the alarm activated?
c If the alarm was activated, what is the probability that there actually was an intruder?
d If the alarm was not activated, what is the probability that there actually was an intruder?
e What was the percentage of accurate results over the test period?
f Comment on the overall performance of the radar detection system.

The information below is to be used in questions 5 to 7.

A test for a medical disease does not always produce the correct result. A positive test indicates that the patient has the condition. The table indicates the results of a trial on a number of patients who were known to either have the disease or known not to have the disease.

	Test results		Total
	Accurate	Not accurate	
With disease	57	3	60
Without disease	486	54	540
Total	543	57	

5 MC The overall accuracy of the test is:

A 90% B 90.5%
C 92.5% D 95%

6 MC Based on the table, what is the probability that a patient who has the disease has it detected by the test?

A 90% B 90.5%
C 92.5% D 95%

7 MC Which of the following statements is correct?

A The test has a greater accuracy with positive tests than with negative tests.
B The test has a greater accuracy with negative tests than with positive tests.
C The test is equally accurate with positive and negative test results.
D There is insufficient information to compare positive and negative test results.

8 Airport scanning equipment is tested by scanning 200 pieces of luggage.

- Prohibited items were placed in 50 bags and the scanning equipment detected 48 of them.
- The equipment detected prohibited items in five bags that did not have any forbidden items in them.

a Use the below information to complete the two-way table below.

	Test results		Total
	Accurate	Not accurate	
Bags with prohibited items			
Bags with no prohibited items			
Total			

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b Use the table to answer the following:
- **i** What percentage of bags with prohibited items were detected?
- **ii** What was the percentage of false positives among the bags that had no prohibited items?
- **iii** What is the probability of prohibited items passing through the scanning equipment undetected?
- **iv** What is the overall percentage accuracy of the scanning equipment?

Further development

9 At a dog show there were 400 dogs. Twenty per cent were purebred and the rest were mixed breeds. Half the dogs were male and half were female. There were 35 male purebred dogs.

a Copy and complete the table below.

	Purebred	Mixed breed	Total
Male	35		
Female			
			400

b Find the probability that a dog chosen at random was a mixed-breed female.

10 The post office trials a new piece of scanning equipment to prevent dangerous substances being sent. To be accepted the equipment must be trialled on 500 items, 20% of which must contain dangerous substances and must successfully detect 99% of the dangerous items. The equipment must produce no more than 4% false positives.

a Complete the table below to show the minimum standard for the equipment to be accepted.

	Positive	Negative	Total
Contains dangerous substance			
Does not contain dangerous substances			
Total			500

b If these minimum standards are obtained, make a statement about the overall percentage accuracy of the equipment.

11 The two-way table below shows the results of a survey taken from among the crowd at a football match.

	Smoker	Non-smoker	Total
Male	72	254	
Female			
Total	115		500

a Copy and complete the table.
b A person is chosen at random from the crowd. Find the probability that the person chosen is a female non-smoker.
c If 20 people are chosen from the crowd, what is the expected number of male smokers?

12 The two-way table below shows the relative frequency of airport scanning equipment activating. An estimated 15 000 people pass through the scanners each day.

	Alarm activated	Alarm not activated	Total
Carrying banned items	0.05	0.02	0.07
Not carrying banned items	0.1	0.83	0.93
Total	0.15	0.85	1.00

a Find the number of times that the alarm is activated during the day.
b Find an estimated number of false alarms per day.

13 The two-way table below shows the results of a medical test for the flu.

	Positive	Negative	Total
With flu	12	2	
Without flu			286
Total		235	

a Copy and complete the table.
b Explain the meaning of the term *false positive* in the context of this question.

14 The two way table below shows the likelihood of a child getting measles compared to the immunisation rates for the disease.

	Had measles	Never had measles	Total
Immunised	2	398	400
Not immunised	45	55	100
Total	47	453	500

a Bella has been immunised against measles. Find the probability that Bella gets measles during childhood.
b Make a statement about benefits, or otherwise, of immunisation against measles. Use the figures in this table to support your opinion.

SUMMARY

Expected outcomes

- The expected number of times that an event will occur in a number of trials is calculated by multiplying the number of trials by the probability of that event occurring.
- The expected number of outcomes is the average number of times that the event is expected to occur. It does not mean this is the number of times the event will occur.

Financial expectation

- Financial expectation is the average financial position at the end of a situation where either a profit or loss will be made.
- The financial expectation is calculated by multiplying each possible financial outcome by the probability of that outcome and then adding the results together.

Two-way tables

- A two-way table is used to display the results of a test and assesses the accuracy of such a test.
- The table can be used to calculate the overall probability of the test achieving its objectives.

CHAPTER REVIEW

MULTIPLE CHOICE

1 A bag contains 3 red marbles, 13 blue marbles and 4 yellow marbles. A marble is chosen from the bag and then replaced in the bag. In 90 selections, the expected number of blue marbles selected is:

A 13 **B** 20
C 58.5 **D** 59

2 A game is played where the player tosses four coins in the air. If all four coins have the same face up, the player wins $6. Otherwise the player loses $1. The financial expectation from this game is:

A –$1.00 **B** –$0.125
C $0.125 **D** $6.00

3 The two-way table below shows the results of a trial on new metal detectors for aircraft. The metal detector scans a piece of hand luggage and lights up if metal is found.

	Test results		
	Accurate	Not accurate	Total
With metal	9	1	10
Without metal	87	3	90
Total	96	4	

Based on the above results, the probability of metal going undetected in a piece of hand luggage is:

A $\frac{1}{10}$ **B** $\frac{1}{4}$ **C** $\frac{3}{4}$ **D** $\frac{9}{10}$

SHORT ANSWER

1 Thirty-six coins are tossed in the air. Calculate the expected number of coins landing Heads.

2 A die is rolled 60 times. Calculate the expected number of:

a 6s **b** even numbers
c numbers less than 3.

3 A card is chosen from a standard deck, noted and replaced in the deck. In 100 trials, calculate (where necessary, correct to 2 decimal places) the expected number of:

a red cards **b** spades
c aces **d** court cards
e black jacks.

4 Two dice are rolled. The score in each roll is the total of the two dice. In 90 rolls of the dice, calculate the expected number of:

a twos **b** sevens
c tens **d** doubles
e totals greater than 8.

5 In a game, three coins are tossed in the air. In 100 tosses of the coins, on how many occasions would you expect the coins to land:

a three Heads?
b two Tails and one Head?
c more Heads than Tails?

6 Two-digit numbers are formed using the digits 2, 4, 7 and 8, and no digit may be repeated. If 60 such numbers are formed, how many numbers can be expected to be:

a 47? **b** even?

c less than 40?

7 Alex bets $10 on the toss of a coin. He calls Heads. If the coin lands Heads, Alex wins $10; if it lands Tails, he loses $10. What is his financial expectation?

8 A bag contains 10 marbles, each with an amount of money written on it. Five marbles have $1 written on them, two have $2 written on them and the others have $5, $10 and $20 written on them. A player pays $5 to draw a marble from the bag and the marble with the amount of money written on it is returned. Calculate the financial expectation from this game.

9 Explain the difference between a positive and negative financial expectation.

10 A roulette wheel is spun. Carly bets $1 on number 25. If 25 is the number spun, Carly will win $35 and have her $1 returned; if not, she will lose $1. Calculate the financial expectation from this game.

11 Jason plays a game where he rolls two dice. If he rolls a total greater than 9, he wins $5; otherwise, he loses $1. Calculate the financial expectation from this game.

12 A bag contains 20 marbles of which 10 are black, 9 are white and 1 is red. Kerry draws a marble from the bag at random. If a black marble represents a $5 loss, a white marble a $4 gain and a red marble a $20 gain, calculate the financial expectation from this game.

13 Over the past 15 years the share price of PHB has risen by $4 in 12 of the years, fallen by $5 in two years and fallen by $10 in the others. If I buy shares in PHB, what would my financial expectation be for the coming year?

14 A medical test screens 200 people for a virus. A positive test result indicates that the patient has the virus.

- Of 50 people known to have the virus, the test produced 48 positive results.
- Of the remainder who were known not to have the virus, the test produced one positive result.

Use the above information to complete the table below.

	Test results		
	Accurate	Not accurate	Total
With virus			
Without virus			
Total			

15 The results of a lie detector test are given below.
- Of 80 people known to be telling the truth, the lie detector indicates that three are lying.
- Of 20 people known to be lying, the lie detector indicates that 17 are lying.

Display this information in a two-way table.

16 Below are the results of a test screening for a disease. A positive test indicates that the patient has the disease.

	Test results		Total
	Accurate	Not accurate	
With disease	18	2	20
Without disease	108	12	120
Total	126	14	

a How many people were tested for the disease?
b How many positive test results were recorded?
c What percentage of those people with the disease were correctly diagnosed by the test?
d If a person without the disease is chosen at random, what is the probability that they returned a positive test?

17 A reading test for people with dyslexia is given and the results are shown in the two-way table below.

	Test results		Total
	Accurate	Not accurate	
With dyslexia	39	1	40
Without dyslexia	85	5	90
Total	124	6	

a How many people were tested?
b What percentage of people tested positive to dyslexia?
c Based on the above results, if a person with dyslexia takes the test, what is the probability that they will be accurately diagnosed?

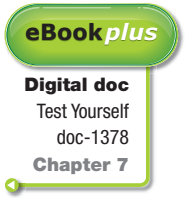
EXTENDED RESPONSE

1 A game is played where two dice are rolled.
a Calculate the probability of rolling a total of 7.
b How many times would you expect to roll a total of 7 in 90 rolls of two dice?
c Calculate the probability of rolling a total of 11.
d Xiao plays a game where he wins $3 for rolling a total of 7 and $7 for rolling a total of 11. Otherwise he loses $1. Calculate the financial expectation for this game.

2 A medical test for a disease does not always give the correct result. A positive test indicates that the patient has the disease. The two-way table below shows the results of a new screening test for the disease. It was tested on a group of people, some of whom were known to be suffering from the disease, some of whom were not.

	Test results		Total
	Accurate	Not accurate	
With disease	28	2	30
Without disease	164	6	170
Total	192	8	

a How many people were tested for the disease?
b What percentage of the results were accurate?
c How many patients tested positive to the disease?
d What percentage of patients with the disease were correctly diagnosed by the new test?
e Based on the above results, what is the probability that a patient with the disease will have the disease detected by this test?



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- SkillSHEET 7.2 (doc-1373): Tree diagrams.
- SkillSHEET 7.3 (doc-1374): Probability trees.

7A Expected outcomes

Tutorial
- WE3 int-2426: Perform a calculation of probability. *(page 228)*

Digital docs
- SkillSHEET 7.1 (doc-1372): Single event probability. *(page 228)*
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7B Financial expectation

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7C Two-way tables

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- Spreadsheet (doc-1376): Two-way frequency tables. *(page 237)*
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